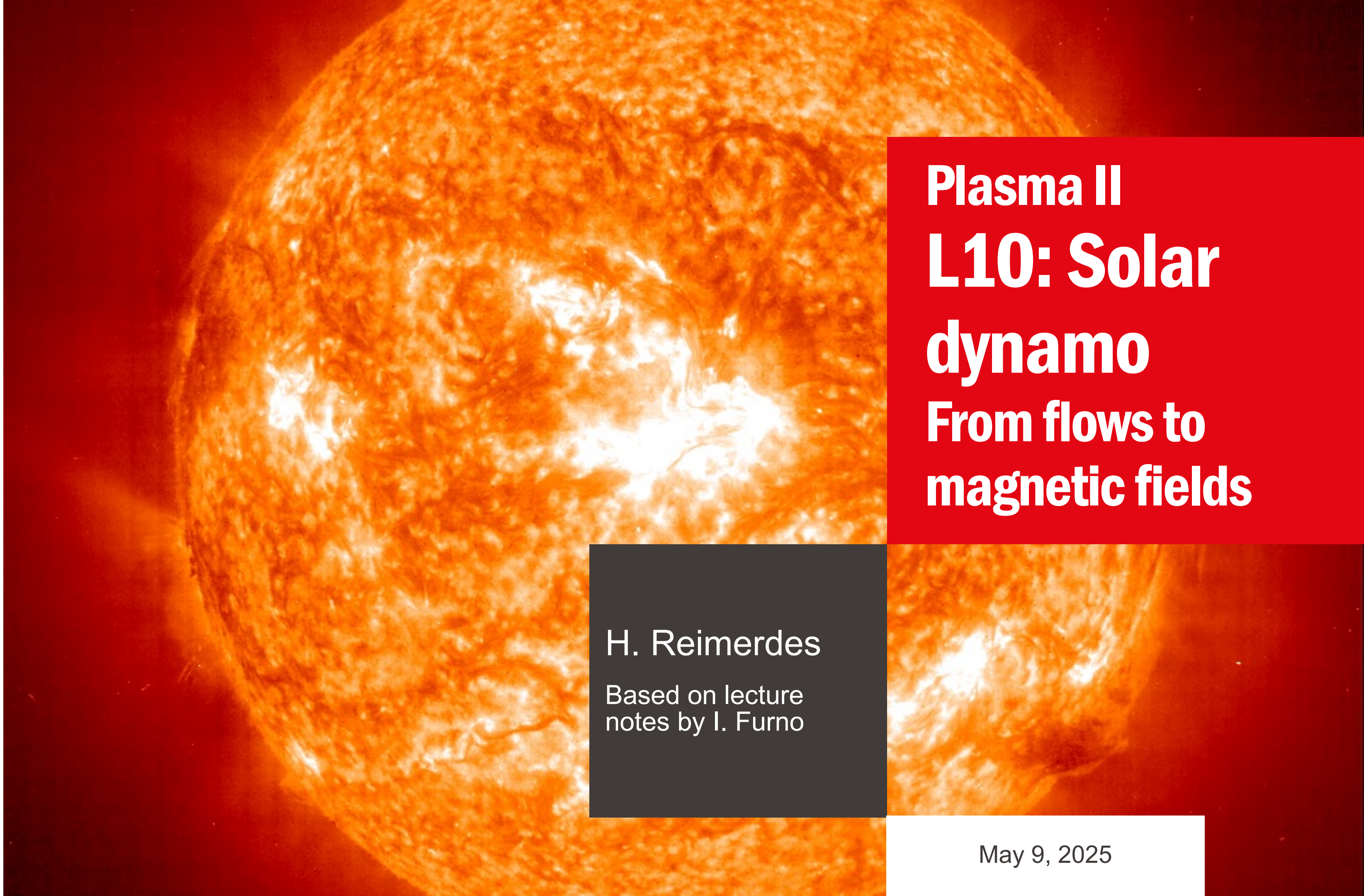


Plasma II
L10: Solar dynamo
From flows to magnetic fields

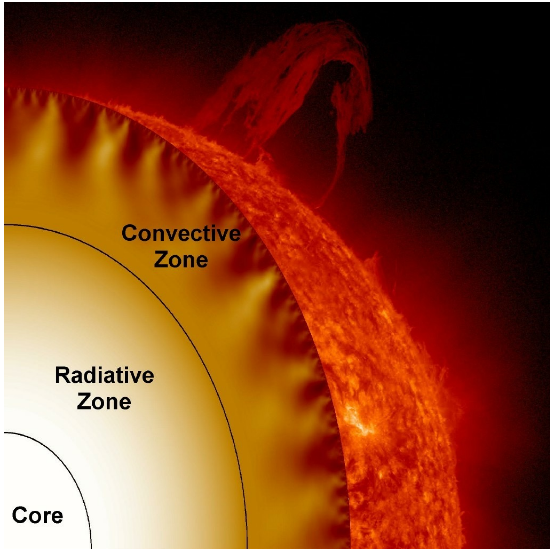
H. Reimerdes

Based on lecture notes by I. Furno

May 9, 2025

Swiss Plasma Center

Content of astrophysics module



- The sun's nuclear energy source
- Transport processes

} L9

- The structure of its magnetic field
- The solar dynamo

} L10

- Magnetic reconnection
- The heliosphere
- Solar wind

} L11

➢ See also EPFL MOOC "Plasma physics: Applications" #4c-d
https://learning.edx.org/course/course-v1:EPFLx+PlasmaApplicationX+1T_2018/home

- N. Meyer-Vernet, "Basics of the solar wind", Cambridge Atmospheric and Space Science Series, Section 3

Sunspots: definition and an early history

- **Sunspots**: dark patches on the Sun
  - Size can be comparable to earth
  - Last for days to months
  - Rotate with the rotation of the sun (~27 day period)
  - Follow a very particular pattern
- Earliest recording from China ~2000 years ago
- The first European observation in 1610 made by Galileo Galilei with his telescope

Courtesy of SOHO (ESA & NASA)

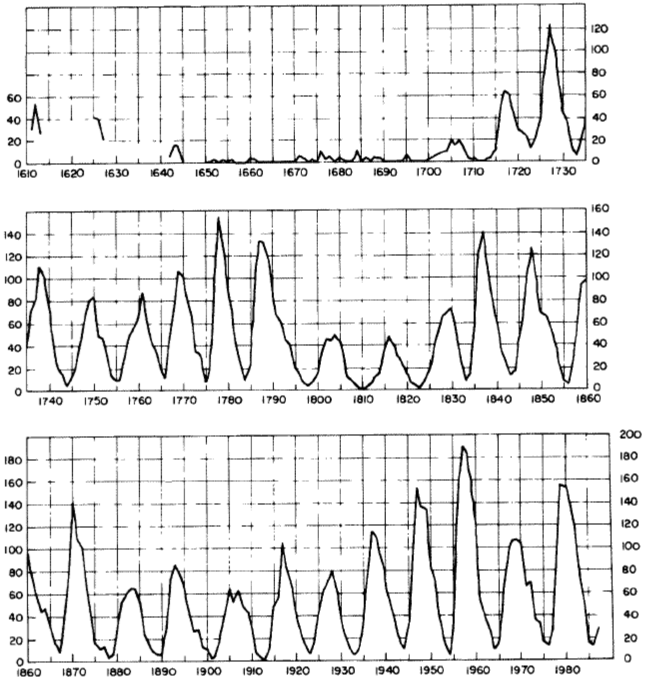
EPFL

Sunspot records

- Sunspots behavior has been reconstructed to the beginning of 17th century
- Reliable records since 1849 due to the tabulation of **J.R. Wolf** in Zurich
- *Relative sunspot number*

$$S_n = k(10g + s)$$

  - s: Number of distinct spots
  - g: Number of recognizable spot groups
  - k: Factor to adjust for differences between telescopes and observers



- Cycle with period varying from 8 to 15 years (average of ~11 years)

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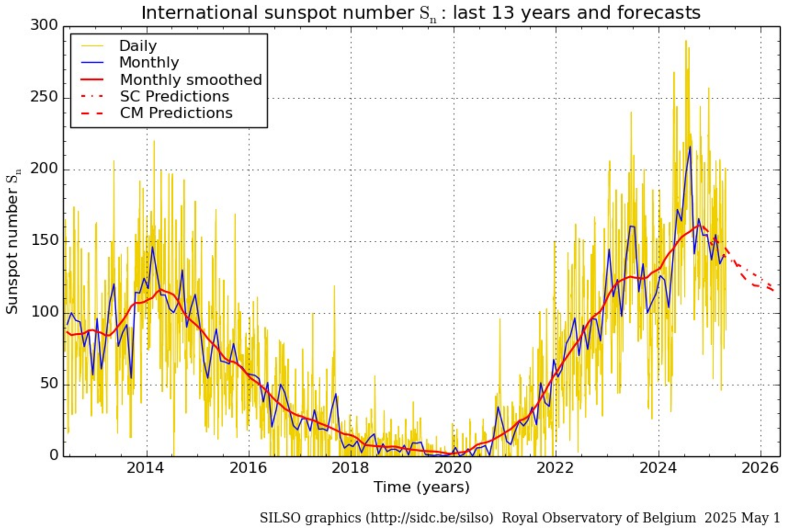

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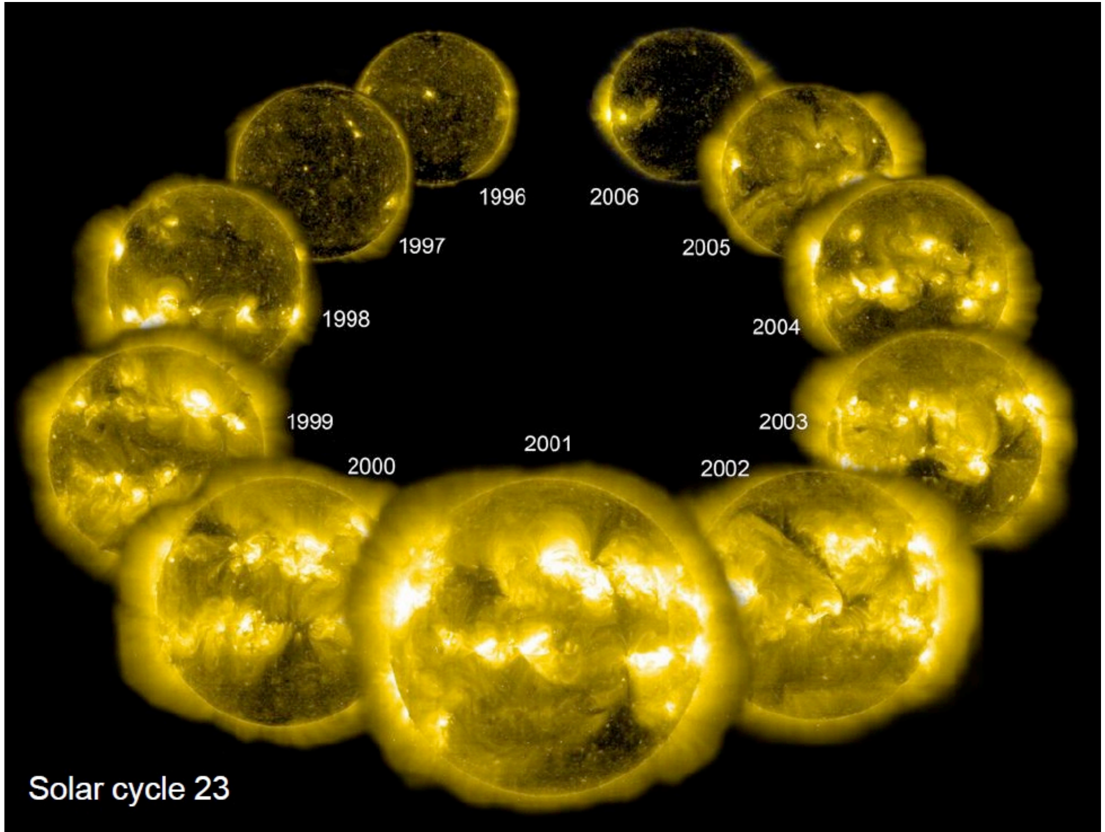
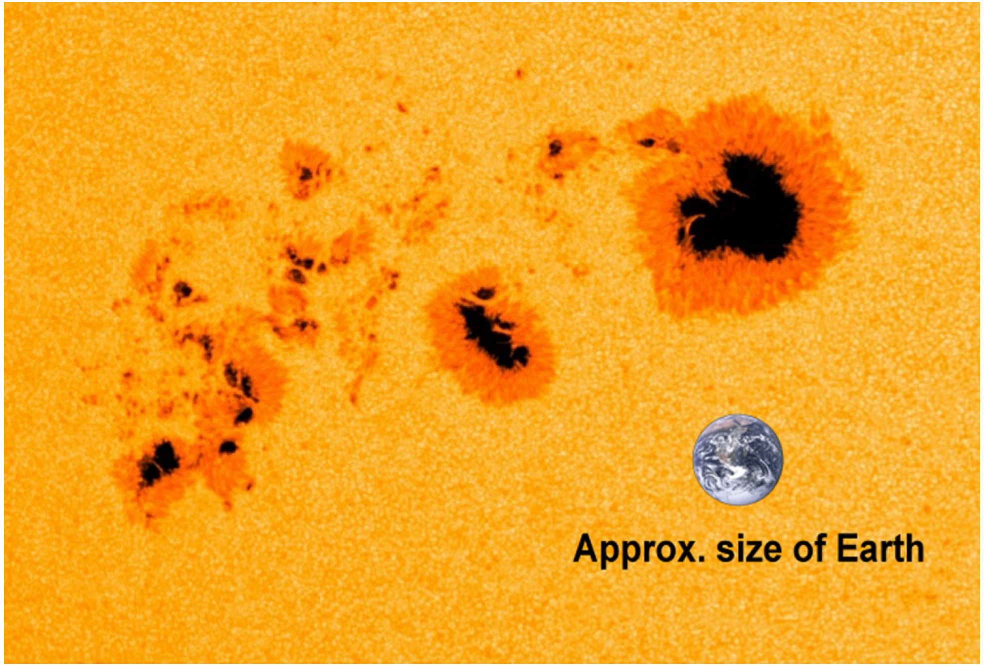
Sunspots: 11 year Solar cycle



A few properties of sunspots

- Size can be comparable with the earth
- Consist of two distinct regions
  - Dark *umbra*
  - Surrounded by brighter *penumbra*
  - Umbra is colder than its surroundings by about 2000K
- They often appear in pairs



Source: NASA

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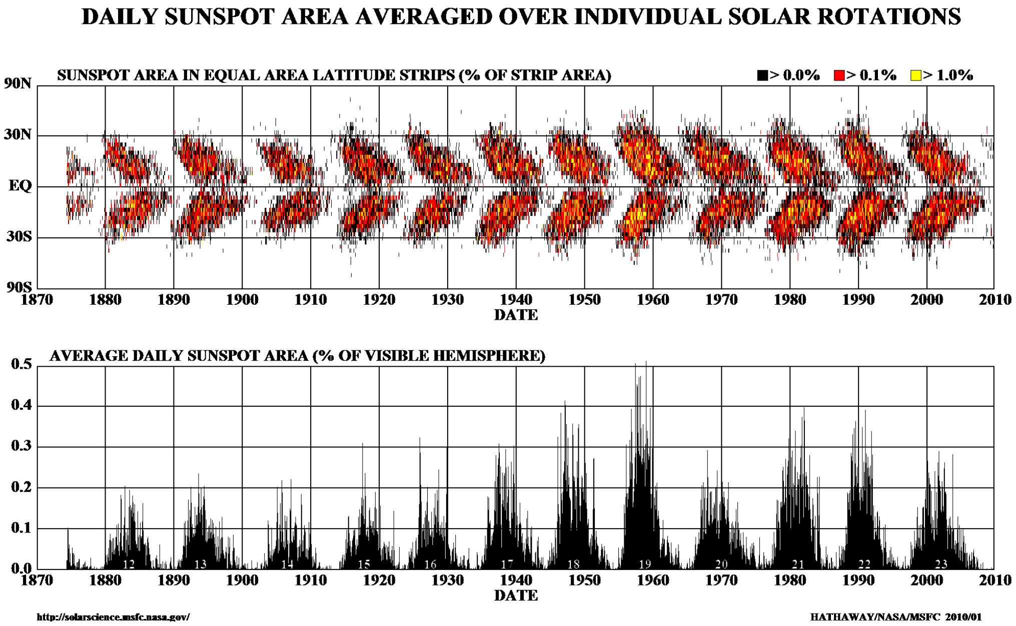
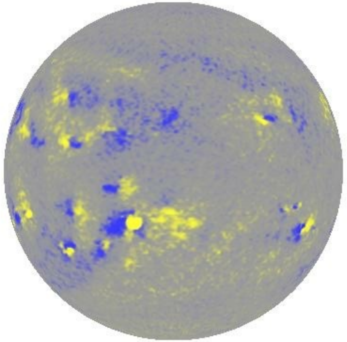
Solar latitude of sunspots varies within the solar cycle

- Sunspots drifts towards the equator as the solar cycle progresses (**Spörer's law**)
- Illustrated in **Maunder's butterfly diagram**

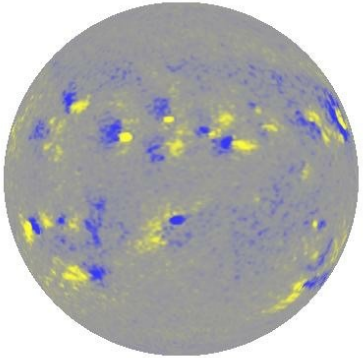


Magnetic fields in sunspots

Cycle 22 -1989 August 02

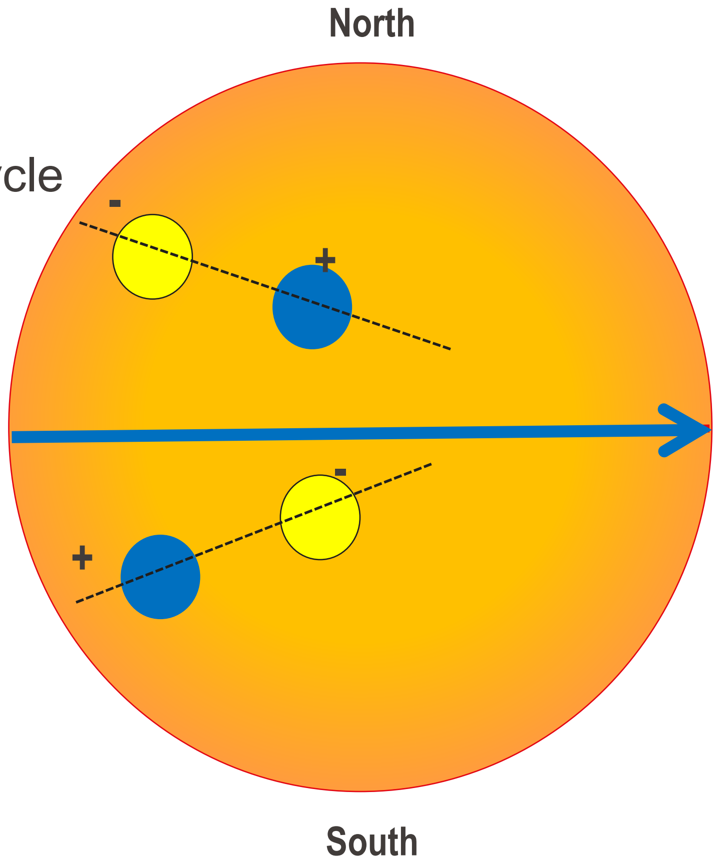


Cycle 23 -2000 June 26

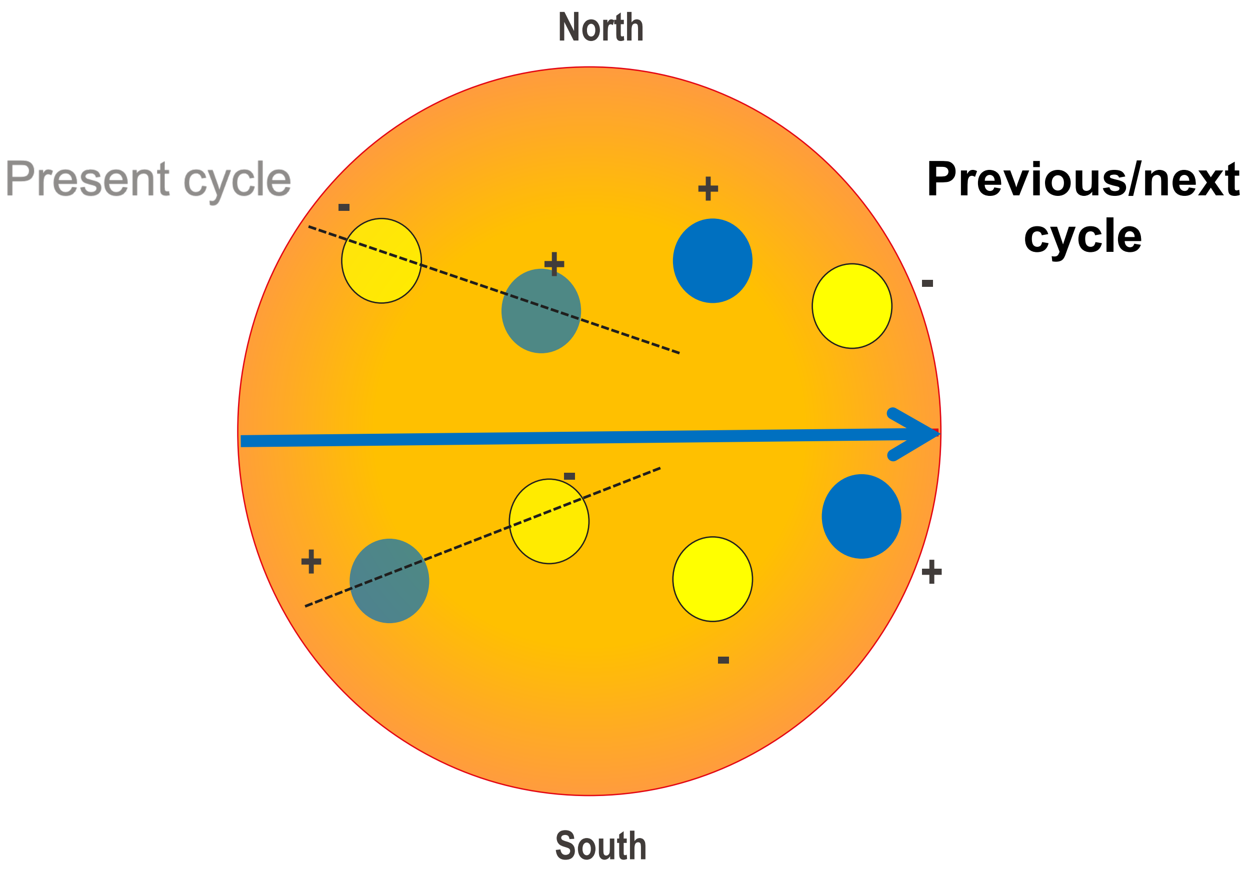


- 1908: George Ellery Hale performed first measurements by Zeeman splitting in sunspots → B ~ 0.1-0.3 T
- 1919: **Hale's polarity law**: "…the preceding and following spots of binary groups, with few exceptions, are of opposite polarity, and … corresponding spots of such groups in the Northern and Southern hemispheres are also of opposite signs. Furthermore, the spots of the present cycle are opposite in polarity to those of the last cycle."
- **Joy's law**: the preceding spot in a bipolar group tends to be closer to the solar equator in both hemispheres

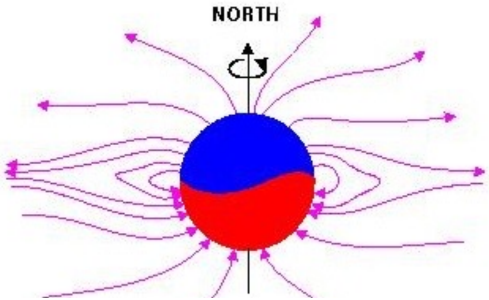
Hale's and Joy's laws illustrated



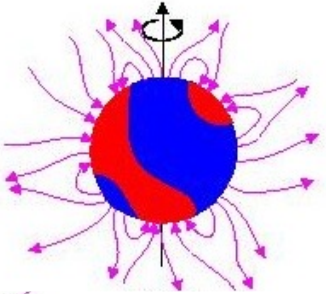
Hale’s and Joy’s laws illustrated



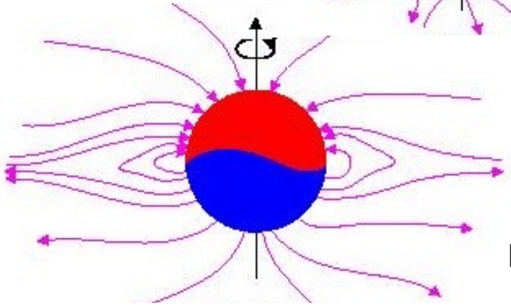
Sun’s global magnetic field structure



Magnetic field at solar minimum



Magnetic field at solar maximum



Magnetic field at the next solar minimum

- Progress in diagnostics allowed to measure much weaker magnetic fields near the poles of the Sun (~1955)
- The polar field is of the order of 10G and it reverses as well at the time of a solar maximum activity
- Polar fields are at their peak near sunspot minimum
  - The magnetic field structure is close to a dipole

Summary - Properties of sunspots

- The 11–year cycle is accompanied by regular oscillations in the latitude distributions of spots and in the polarity of their fields
- The latitude band occupied by sunspots drifts towards the equator during a solar cycle → Spörer’s law / Maunder butterfly diagram
  - First spots of a new cycle centered around 25-30 deg. latitude, while last spots found 20 deg. closer to the equator
- The magnetic field polarity of sunspot pairs was studied by Hale
  - Preceding and following spots have opposite polarity
  - Pattern is reversed in the other hemisphere
  - Polarity of the preceding sun spot changes in a new cycle
  - Joy’s addition: preceding spot tends to be closer to the equator
- The magnetic field at the poles is strongest during the sunspot minimum and has a dipole structure
  - Field much weaker than field of sunspots
  - Direction of di-pole reverses with sunspot cycle → 22 year period

Is a primordial magnetic field at the origin of the Sun's magnetic field?

- Estimate time scales for changes in the magnetic field

Reminder: Magnetic diffusion equation

Is a primordial magnetic field at the origin of the Sun's magnetic field?

- Estimate time scales for changes in the magnetic field

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\frac{\nabla^2 \mathbf{B}}{\mu_0 \sigma}} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\tau_{\mathrm{res}} \sim \sigma \mu_0 L^2$$

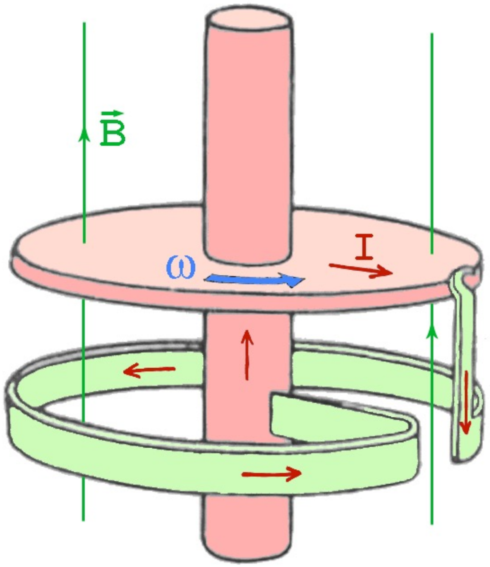
  - Estimate conductivity $\sigma = ne^2/(m_e \nu_{e/i}) \approx 6\times10^{-4}\, T^{3/2} (\mathrm{K}^{3/2} \Omega m)^{-1}$ using the Sun's average temperature $\langle T \rangle \approx 2.3\times10^6 K$ yields $\sigma \sim 10^6 \Omega^{-1} \mathrm{m}^{-1}$
  - With $L = L_\odot \approx 7\times10^8 \mathrm{m}$ the resistive diffusion time is $\boldsymbol{\tau_{res} \sim 10^{18} \mathrm{s}}$ or $\sim 10^{11} \mathrm{years}$

→ Other processes are needed: **DYNAMO!**

Dynamo theory

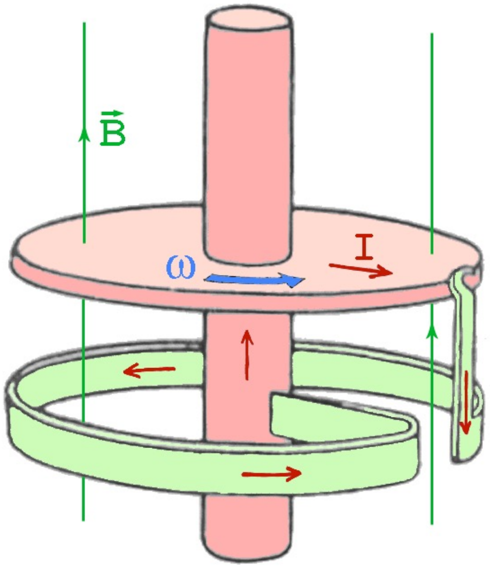
- Sir Joseph Larmor, Report of the British Association for the Advancement of Science 87th meeting, 1919, "*How could a Rotating Body such as the Sun become a Magnet?*"

- **Dynamo**: process of magnetic field generation by the inductive action of a conductive fluid
  - Conversion of mechanical energy into magnetic energy by stretching and twisting magnetic field lines



Homopolar disk dynamo (Bullard 1955)

Dynamo theory



Homopolar disk dynamo (Bullard 1955)

Magnetic induction equation and Magnetic Reynolds number

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

Minimum R_m for dynamo

- If the R_m is small, magnetic diffusion dominates and no dynamo is possible
- The magnetic energy equation is formed by taking the scalar product of the induction equation with $\mathbf{B}/\mu_0$

$$\frac{\partial}{\partial t}\int \frac{\mathbf{B}^2}{2\mu_0} dV = \int \frac{1}{\mu_0\sigma} \underbrace{\nabla^2\mathbf{B}}_{-\nabla\times(\nabla\times\mathbf{B})} \cdot \mathbf{B}\, dV + \int \nabla\times(\mathbf{u}\times\mathbf{B}) \cdot \frac{\mathbf{B}}{\mu_0} dV$$

- Identify magnetic energy E_M and rearrange triple products

$$\mu_0 \frac{\partial E_M}{\partial t} = -\frac{1}{\mu_0\sigma}\int |\nabla\times\mathbf{B}|^2 dV + \int (\nabla\times\mathbf{B}) \cdot (\mathbf{u}\times\mathbf{B}) dV$$

  - The last term is a vector triple product, and since $\mathbf{a} \cdot (\mathbf{b}\times\mathbf{c}) \leq |\mathbf{a}||\mathbf{b}||\mathbf{c}|$ with equality only when all three vectors are perpendicular

$$\int (\nabla\times\mathbf{B}) \cdot (\mathbf{u}\times\mathbf{B}) dV \leq u_{\max} \left(\int |\nabla\times\mathbf{B}|^2 dV\right)^{1/2} \left(\int |\mathbf{B}|^2 dV\right)^{1/2}$$

Minimum R_m for dynamo (cont.)

- General property of a divergence free fields confined in a sphere of radius *a*

$$\int |\nabla\times\mathbf{B}|^2 dV \geq \frac{\pi^2}{a^2}\int |\mathbf{B}|^2 dV$$

- This simplifies the inequality on the previous page

$$\int (\nabla\times\mathbf{B})\cdot(\mathbf{u}\times\mathbf{B})dV \leq u_{\max}\left(\int |\nabla\times\mathbf{B}|^2 dV\right)^{1/2}\left(\int |\mathbf{B}|^2 dV\right)^{1/2}$$

$$\leq \frac{a u_{\max}}{\pi}\int |\nabla\times\mathbf{B}|^2 dV$$

- Hence

$$\mu_0\frac{\partial E_M}{\partial t} \leq \left(\frac{a u_{\max}}{\pi} - \frac{1}{\mu_0\sigma}\right)\int |\nabla\times\mathbf{B}|^2 dV$$

➢ An increase of the magnetic energy (**dynamo**) requires $R_m = \mu_0\sigma a u_{\max} \geq \pi$

The magnetic Reynolds number in the Sun

$$R_m = \mu_0 \sigma L u$$

Theorems for kinematic dynamo

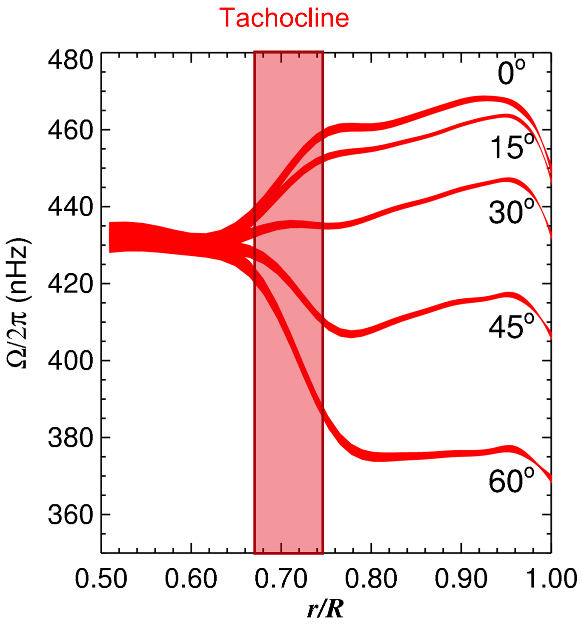
Kinematic dynamo: fluid flow capable of amplifying a magnetic field

- **Theorem 1**: In Cartesian coordinates (x; y; z) no field independent of z which vanishes at infinity can be maintained by dynamo action → it is impossible to generate a 2D dynamo field
- **Theorem 2**: No dynamo can be maintained by a planar flow (u_x; u_y; 0)
  - No restriction is placed on whether the field is 2D or not in this theorem
- **Theorem 3** (*Cowling's theorem*): An axisymmetric magnetic field vanishing at infinity cannot be maintained by dynamo action
- **Theorem 4**: A purely toroidal flow cannot maintain a dynamo

➢ **Dynamos with a high degree of symmetry do not work!**
  - Hence, a successful dynamo has to be complicated … and so is its analysis

- Many astrophysical applications: galaxies, stars, planets, earth, accretion disks

Solar rotation

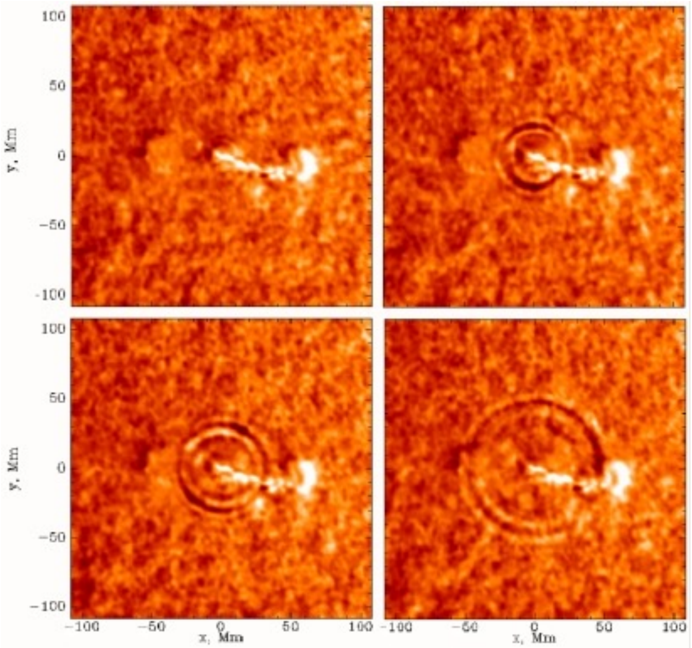


Howe, R. et al. 2000, *Dynamic variations at the base of the solar convection zone*, Science 287, 2456.

- Equatorial regions of the Sun rotate faster than polar ones
- Most radial variation takes place in a thin layer at the base of the *convection zone*, where the angular velocity adjusts to that of the interior (*tachocline region*)
- The interior appears to rotate approximately as a rigid body
  - Angular velocity of interior between values at equator and poles

How do we look 'inside' the Sun? → Helioseismology

Sun quake



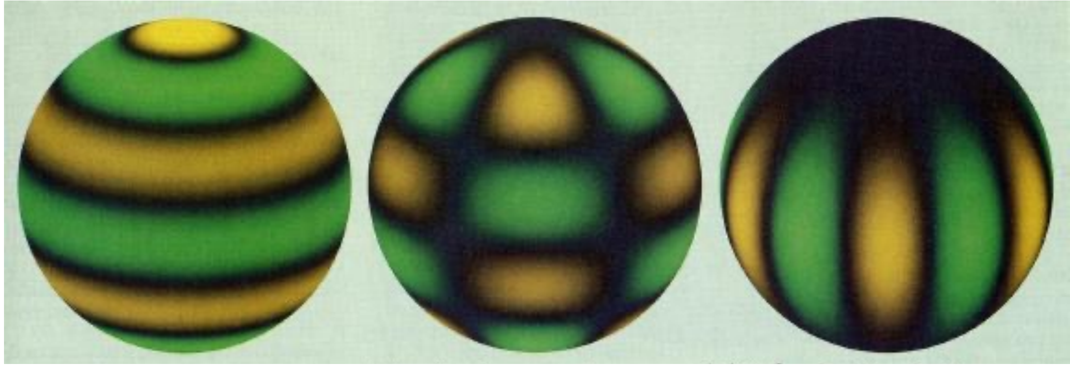
Kosovichev & Zharkova, 1998, Nature

- Seismology studies seismic waves to derive Earth's interior structures
- *Helioseismology* is a science to study solar interior properties by studying helioseismic waves

Global Helioseismology

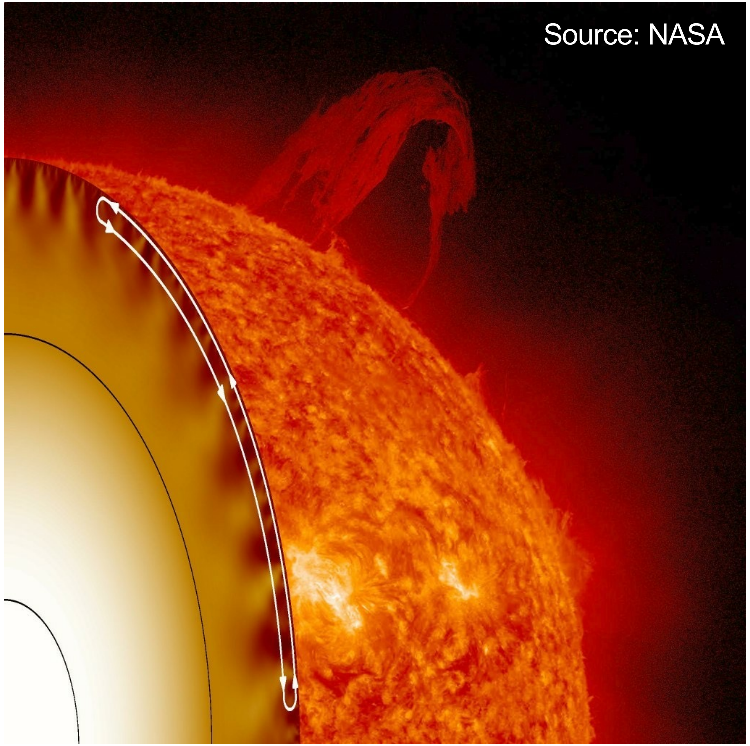
- **Data from space** – in particular from three instruments on the SOHO satellite: SOI-MDI, GOLF, VIRGO and **data from Earth observatories** measure mode properties of global solar oscialltions: frequencies, eigenfunctions / spherical harmonics

- Frequencies ω_{nlm} depend on conditions in solar interior that determine wave propagation



➢ Inversion provides maps of density ρ and rotation Ω

Solar flows



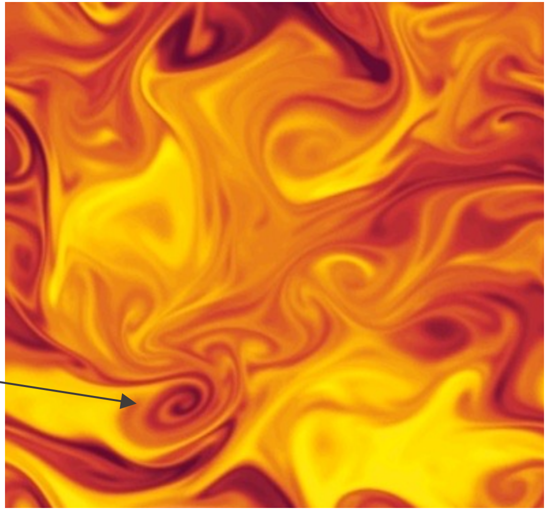
- *Meridional circulation*: flow along meridian lines from the equator towards the poles at the surface and from the poles to the equator below the surface
  - At the surface the flow is slow ~20m/s
  - The return flow towards the equator is much slower 1-2m/s
    - This slow return flow would carry material from the mid-latitudes to the equator in ~11year (!)

Effect of sheared plasma flow on magnetic flux tubes

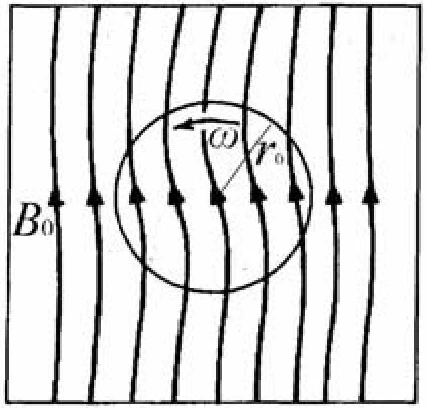
- Effect of sheared plasma flow

How does turbulence act on the B-field?

Turbulent eddy



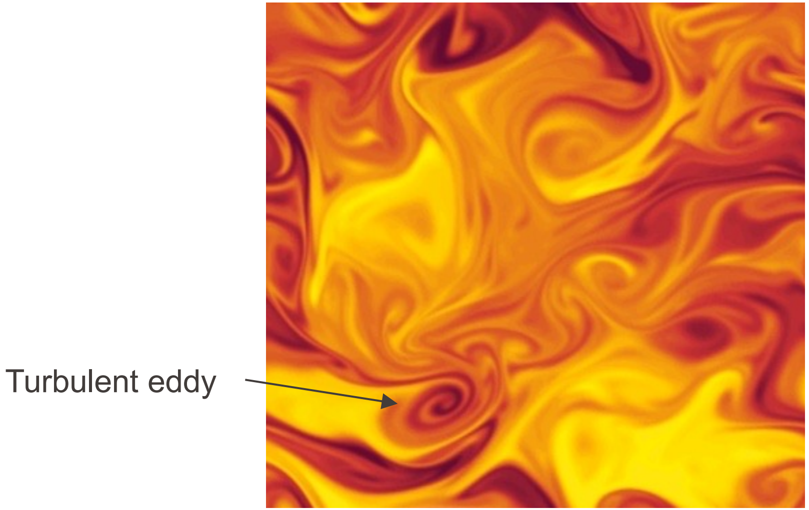
Simplified picture of a turbulent eddy



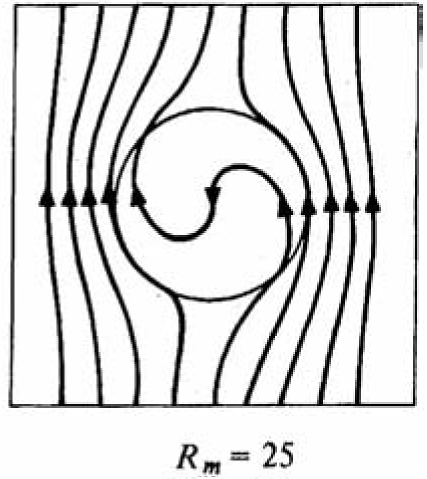
$$\boldsymbol{R_m = 3}$$

$$R_m = \mu_0 \sigma L u = \mu_0 \sigma \omega r_0^2$$

How does turbulence act on the B-field?



Simplified picture of a turbulent eddy



$$R_m = \mu_0 \sigma L u = \mu_0 \sigma \omega r_0^2$$

- This magnetic field expulsion is related to the skin effect in electrical engineering
  - Skin depth $\delta = 2/\sqrt{\mu_0 \sigma \omega} \;\; = r_0 / R_m^{1/2}$

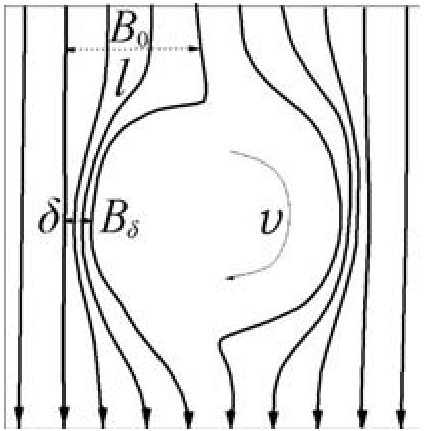
Expelling and concentrating the B-field

- Conservation of magnetic flux

$$B_0 r_0 = B_\delta \delta$$

➢ Field in the boundary layer of eddy is amplified

$$B_\delta = B_0 \frac{r_0}{\delta} = B_0 \sqrt{R_m}$$



Buoyancy of magnetic flux tubes

- Preserve total pressure

$$P_{\text{int}}(z) + \frac{B^2(z)}{2\mu_0} = P_{\text{ext}}(z)$$

➢ A large magnetic field amplitude requires that $P_{\text{int}} < P_{\text{ext}}$ and gives rise to buoyancy → flux tubes rise towards the surface (**see Exercise 1**)

➢ The pressure generally decreases towards the sun's surface (**see L9**)

$$\frac{dP_{\text{int}}(z)}{dz} = -\rho_{\text{int}} g_{\odot} \qquad \frac{dP_{\text{ext}}(z)}{dz} = -\rho_{\text{ext}} g_{\odot}$$

- Derivative of pressure balance with respect to z yields

$$B\frac{dB}{dz} = g_{\odot}\mu_0(\rho_{\text{int}} - \rho_{\text{ext}}) < 0$$

➢ Magnetic field decreases towards the surface and field lines fan out to conserve magnetic flux

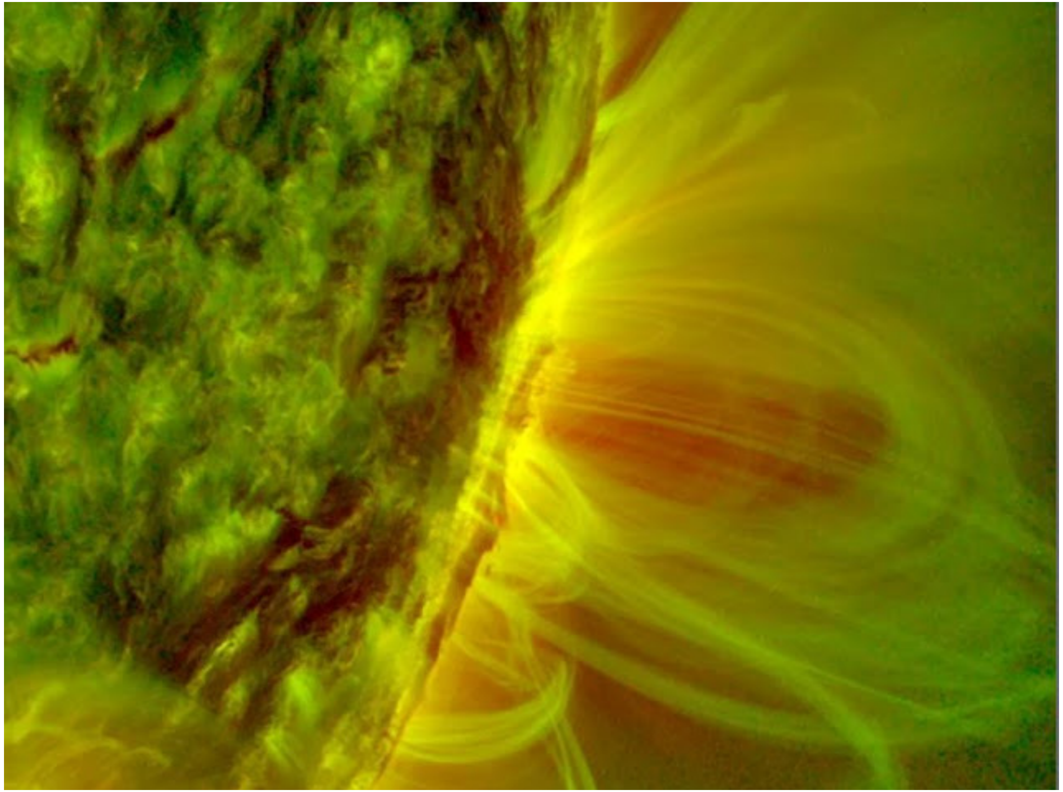
Expelling and concentrating the B-field

Summary

- For $R_m \gg 1$, a rotating eddy (← turbulence) excludes the magnetic field from its interior
- Magnetic flux accumulates at the boundaries, forms thin sheets or tubes that are much smaller than the eddy ($\delta \ll r_0$) → magnetic field strength is strongly amplified

➢ The magnetic flux at the solar surface is concentrated into small regions of large magnetic field, at the boundaries of convective structures
  - Plasma can flow along flux tubes, but not across them
  - Magnetic flux tubes are less dense than their surrounding and rise towards the surface

Flux ropes or flux tubes in reality



https://www.youtube.com/watch?v=lHiii7glenE

Summary so far

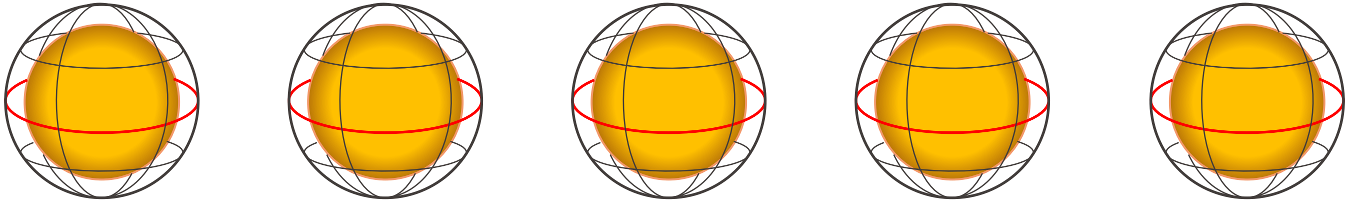
H. Reimerdes

- Magnetic field can be generated by a dynamic dynamo
  - Energy comes from the kinetic energy of the flow
  - A dynamo requires asymmetries
- Global flows
  - Azimuthal rotation of the outer shell differs from the equator to the poles
  - Core rotates rigidly leading to a transition layer with strongly sheared flow
  - Slow meridonal circulation on the Sun's surface
- Turbulent flows
  - Expel magnetic field and creates tubes/sheets of strong magnetic field
  - Tubes are buoyant and rise and or flare
- High R_m → ideal MHD → plasma and field move together

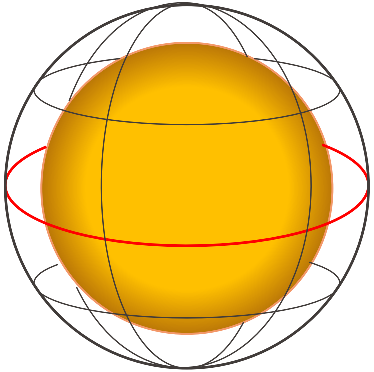
B field inversion: The Ω-effect

H. Reimerdes

- Consequence of the sheared solar rotation

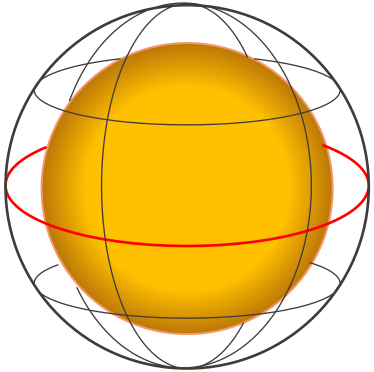


B field inversion: The α-effect (Parker 1955)



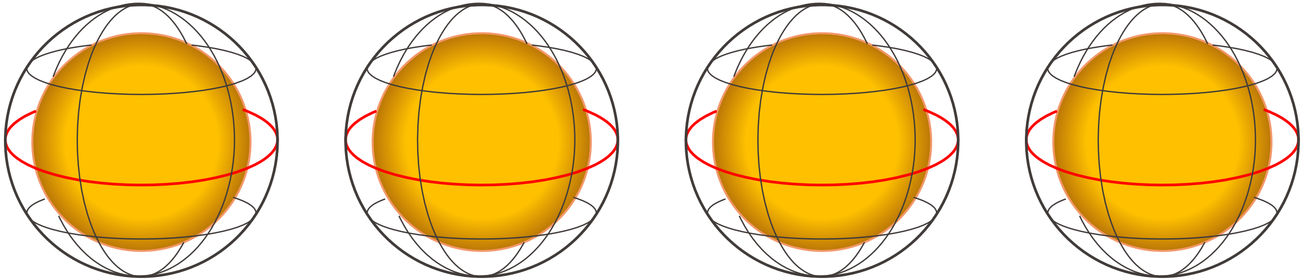
- The original α-effect relies on small scale turbulent motions whose twisting action can transform a toroidal into a poloidal field line
- The turbulent motions must have non-zero fluid helicity → screw-like vortices (cyclones)
- Averaging over many cyclonic events leads to the production of a large-scale poloidal field
- This mechanism is formalised in the *mean-field electrodynamics*

The Babcock-Leighton mechanism



- Developed in the 1960's, it was obscured by mean-field electrodynamics
- If the toroidal magnetic field is too strong, turbulence cannot twist the magnetic field lines thus reducing the dynamo effect → revival of BL mechanism

The Babcock-Leighton mechanism



Babcock's model: a simulation

http://video.ucar.edu/eo/sparkSite/video/ogg/sun_mag_field_rotate_tangle_400x300.ogg